[2] obtained from numerical solutions of integral equations. Agreement between the present results and those of Lin is seen to be good.

A presentation of ϵ_a results for a cone half angle of 5° is made in Fig. 2. The structure of this figure is similar to that of Fig. 1, and the findings are generally similar. As before, the fluctuations exhibited by the partitioning results are appreciably less than are those in the results from the conventional solution method. Also, the partitioning results converge more rapidly.

The findings presented above indicate that the use of the energy partitioning approach may be highly advantageous in problems where a portion of the energy content of a ray bundle is governed by deterministic laws (e.g. geometrical angle factors). Another interesting outcome of the present work is the documentation of the fluctuations experienced by the Monte Carlo results as the number of rays is increased. This suggests that the output corresponding to a specific pre-selected number of ray bundles may not always be a proper representation of the results.

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HEAT TRANSFER ACROSS TURBULENT FALLING FILMS

A. F. MILLS and D. K. CHUNG

School of Engineering and Applied Science, University of California, Los Angeles, California 90024, U.S.A.

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NOMENCLATURE

- g, gravitational acceleration;
- h_c, heat transfer coefficient;
- k, thermal conductivity;
- Pr, Prandtl number;
- Re, Reynolds number $= 4\Gamma/\mu$;
- Sc, Schmidt number;
- u^+, dimensionless velocity $= u/\sqrt{(g\delta)}$;
- y^+, dimensionless coordinate measured from wall $= \dfrac{y\sqrt{(g\delta)}}{\nu}$;
- Γ, flow rate per unit width;
- δ, film thickness;
- δ^+, dimensionless film thickness;
- ε, eddy diffusivity;
- ε^+, $= 1 + \varepsilon/\nu$;
- μ, dynamic viscosity;
- ν, kinematic viscosity;
- ρ, density;
- σ, surface tension.

Subscripts

- D, mass species diffusion;
- M, momentum;
- i, intersection of equations (1) and (4);
- t, turbulent.

Recently Chun and Seban [1] presented the results of an experimental study into heat transfer across evaporating turbulent falling films. They found $h_c \propto Re^{0.4}$, whereas usual analyses e.g. [2,3] based on the conventional hypotheses about turbulent transport that are consistent with pipe flow, predict $h_c \propto Re^{0.2}$ in the limit of high Re (see Fig. 1). Our purpose here is to present an analysis which successfully predicts the Chun–Seban data, and which may also be used for film condensation provided that vapor drag is negligible.

Our starting point is the observation that conventional hypotheses about turbulent transport do apply close to the solid wall, as demonstrated by the electrolytic mass transfer experiments of Iribarne *et al.* [4]. Any one of a number of eddy diffusivity profiles could therefore be used in the near wall region; we choose to use that of van Driest [5],

$$\varepsilon_M^+ = \tfrac{1}{2}\{1 + \sqrt{[1 + 0{\cdot}64 y^{+2}(1 - \exp(-y^+/26))^2]}\}. \quad (1)$$

Next we suggest that the nature of turbulent transport near the interface should be reliably shown by gas absorption experiments. Lamourelle and Sandall [6] measured mass transfer coefficients for gas absorption into a turbulent falling film and deduced that a consistent eddy diffusivity profile was

$$\varepsilon_D = 7{\cdot}33 \times 10^{-6}\, Re^{1{\cdot}678}(\delta - y)^2 \ \mathrm{m^2/s}. \quad (2)$$

or for computational purposes,

$$\varepsilon_D^+ = 1 + 6{\cdot}47 \times 10^{-4} \frac{\rho g^{\frac{1}{3}} \nu^{\frac{4}{3}}}{\sigma} \frac{Re^{1{\cdot}678}}{\delta^{+\frac{4}{3}}} (\delta^+ - y^+)^2 . \quad (4)$$

Accurate specification of the eddy diffusivity in the middle region of the film is not required owing to the low thermal resistance there. Thus the complete eddy diffusivity profile can be obtained by simply running equations (1) and (4) until they intersect, at y_i^+ say. The model of turbulent transport is completed by specifying $Pr_t = 0{\cdot}9$ following recent boundary layer practice, e.g. [8], and $Sc_t = 1{\cdot}0$ in the absence of reliable evidence to the contrary.

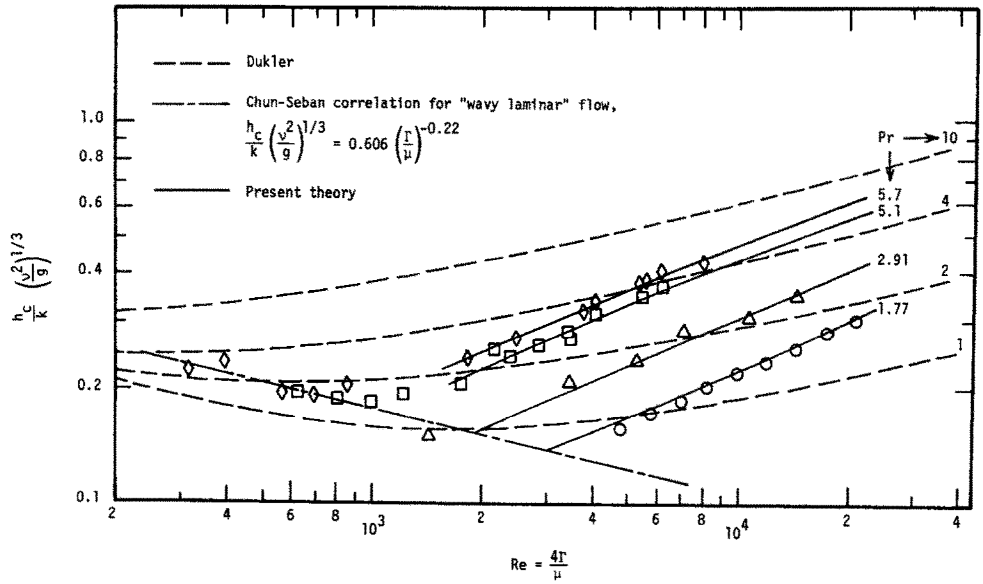

FIG. 1. Local heat transfer coefficient as a function of Reynolds number; comparisons of theory with the experiments of Chun and Seban.

Unfortunately equation (2) is not dimensionless, and was based only on results for water at 25°C. Chun and Seban did obtain a set of evaporation data for water at 28°C, so that equation (2) could be used directly in that case. In order to generalize equation (2) to temperatures other than 25°C, and to liquids other than water, it must be rendered dimensionless in a manner which recognizes the essential physics involved. Following Levich [7] we view the damping of the eddy diffusivity represented by equation (2) as due to surface tension and thus suggest the dimensionless form

$$\varepsilon_D^+ = 1 + 6{\cdot}47 \times 10^{-4} \frac{g\rho}{\sigma} Re^{1{\cdot}678} (\delta - y)^2 , \quad (3)$$

The calculation procedure to yield the heat transfer coefficient invokes the usual Nusselt assumptions of negligible liquid acceleration and thermal convection, and constant properties. Since the shear distribution is linear there results for the velocity profile

$$u^+(y^+) = \int_0^{y^+} \frac{1 - y^+/\delta^+}{\varepsilon_M^+} \mathrm{d}y^+ , \quad (5)$$

and the Reynolds number is defined by

$$Re = \frac{4\Gamma}{\mu} = 4 \int_0^{\delta^+} u^+ \,\mathrm{d}y^+ . \quad (6)$$

Equations (1) and (4) are substituted in equation (5) and starting with nominal values of Re and δ^+, equations (5) and (6) are solved iteratively. The corresponding heat transfer coefficient is obtained by integrating the energy conservation equation;

$$\frac{h_c}{k}\left(\frac{\nu^2}{g}\right)^{\frac{1}{3}} = \frac{Pr(\delta^+)^{\frac{1}{3}}}{F(\delta^+)}, \tag{7}$$

where

$$F(\delta^+) = \int_0^{\delta^+} \frac{dy^+}{\frac{1}{Pr} + \frac{1}{Pr_t}(\varepsilon_M^+ - 1)}. \tag{8}$$

Figure 1 compares our predictions with the Chun–Seban experiments. Data was obtained at four saturation temperatures: 28, 38, 62 and 100°C; the corresponding liquid Prandtl numbers were 5·7, 5·1, 2·91 and 1·77 respectively. The agreement is seen to be excellent. At 28°C we are in effect demonstrating the consistency of the experimental data on which our calculation procedure is based. The good agreement at other temperatures confirms that the scaling with surface tension in equation (3) is appropriate. We note further that (i) y_i^+ varied from 0·3 δ^+ to 0·85 δ^+ as Re increased through the range considered, and (ii) the gas absorption experiments showed the mass transfer coefficient to be proportional to $Re^{0\cdot8}$; thus the result $h_c \propto Re^{0\cdot4}$ is not unexpected despite the fact that the near wall transport model suggests $h_c \propto Re^{0\cdot2}$. Figure 1 of course applies to turbulent film condensation as well. Extension of this work to include the effects of vapor drag must await the availability of appropriate gas absorption experimental data.

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JOULE–THOMSON EFFECTS ON THERMAL ENTRANCE REGION HEAT TRANSFER IN PIPES WITH UNIFORM WALL TEMPERATURE

K. C. CHENG and JENN-WUU OU
Department of Mechanical Engineering, University of Alberta, Edmonton, Alberta, Canada

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NOMENCLATURE

Br, Brinkman number, $\mu U_m^2/k(T_0 - T_w) = PrEc$;
c_p, specific heat at constant pressure;
$h, \bar{h}$, specific enthalpy and local heat transfer coefficient, respectively;
k, thermal conductivity;
p, local axial pressure;
$\bar{q}$, heat flux vector;
R, X, radial and axial coordinates;
R_0, pipe radius;
T, T_0, T_w, gas temperature, uniform gas temperature at thermal entrance and constant wall temperature, respectively;